FUZZY MODELLING OF TUNNEL DIODE CIRCUIT

By

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ABSTRACT

This paper presents the T-S fuzzy model of the tunnel diode circuit which is one of the well-known benchmark in non-linear problem. The non-linear differential equation of tunnel diode is linearized with the help of T-S fuzzy model which consists of a number of linear sub-systems. At the end, it was observed that the over-all fuzzy system is unstable, so the fuzzy controller using Parallel Distributed Compensation (PDC) approach is employed for its stable working.

Keywords: T-S Fuzzy Model (Takagi Sugeno Fuzzy Model), Tunnel Diode, Parallel Distributed Compensation (PDC).

INTRODUCTION

Tunnel diode is a microwave device which is developed by Esaki that is why it is also known as Esaki diode. It consists of a p-n junction in which both the p-side and n-side are heavily doped with impurities and it consists of Negative Differential Resistance (NDR), which was first fabricated in 1958 [1]. Due to this NDR, it can be extended from the range of DC to giga-hertz frequency. The DC characterization of the NDR region of tunnel diode is often an obstacle by parasitic oscillations [2, 3]. Tunnel diode is useful in many circuit applications like microwave amplification circuits, oscillators, multi-vibrators, binary memory, counters, micro-power transmitters and industrial uses [4].

The tunnel diode is very useful in microwave application because it exhibits a negative-resistance characteristic in the region between peak current I_p and valley current I_v. The I–V characteristic of a tunnel diode in shown in Figure 1. The points a and c are stable, but point b is unstable [5].

It is observed from Figure 1 that the tunnel diode has multiple equilibria as a bi-stable circuit. Due to this characteristics of multiple equilibrium states, it has a non-linear differential equation, so it becomes necessary to linearize this equation of the tunnel diode circuit for the purpose of steady state analysis of the circuit.

Fuzzy system is applied in various fields of application like an automatic train operation, an automatic container crane operation, elevator control, nuclear reactor control and motor control [6, 7]. In general, it can be classified as Mamdani type and Takagi-Sugeno (T-S) type. The Mamdani type fuzzy control system is well recognized [8], and the T-S type fuzzy system focus basically on the modeling aspect [9].

The model linearization of T-S fuzzy model can be treated as universal approximators as it is able to approximate any non-linear functions with an arbitrary accuracy [10, 11]. A survey on analysis and design for model based fuzzy control system is given in [12]. In [13], a new method is proposed for fuzzy control of non-linear system and many works are also done for fuzzy modeling and control of non-linear systems [14-17].

In this paper, the T-S fuzzy model for the tunnel diode circuit is developed which has fifth order polynomial equation

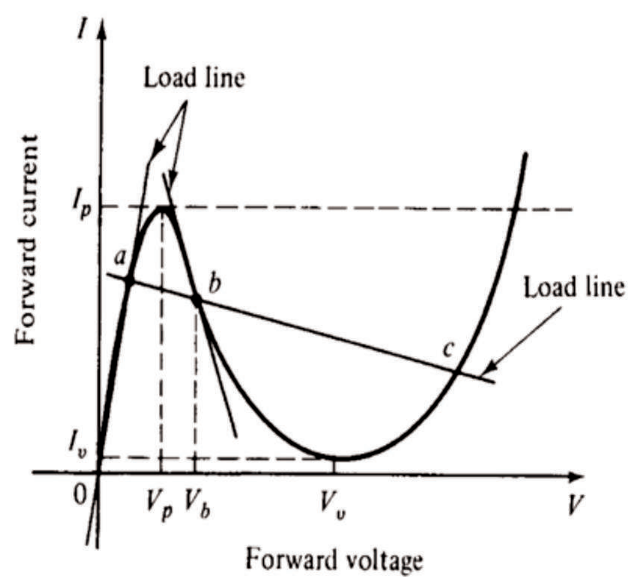


Figure 1. I-V characteristic of a Tunnel Diode with Load Line

representing the characteristics of the tunnel diode. The state space equation of the tunnel diode circuit can be obtained by the state-space technique [18] by employing the circuit parameters as state variables.

In the second section, T-S fuzzy model for non-linear system will be introduced. In the third section, the state space and fuzzy model of tunnel diode circuit will be given. In the fourth section, the simulation result is shown and at last discussion will be done from it.

1. T-S Fuzzy Model for Non-linear System

Let us consider a general non-linear system described by the following equation:

$$\dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{f}(\boldsymbol{x}(\boldsymbol{t}), \boldsymbol{u}(\boldsymbol{t}), \boldsymbol{t}) \tag{1}$$

The equation (1) can be represented by T-S fuzzy model [9] as given below:

Model rule i

IF

$z_1(t)$ is N_{i1} and … … … … $z_p(t)$ is N_{ip};

THEN

$$\dot{\boldsymbol{x}}(\boldsymbol{t}) = \boldsymbol{A}_i\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B}_i\boldsymbol{u}(\boldsymbol{t}) \tag{2}$$

for i=1, 2, 3, ……… r where N_{ij} is the fuzzy set; $x(t)\in R^n$ is the state vector; $u(t)\in R^m$ is the control input; $A_i\in R^{n\times n}$, $B_i\in R^{n\times m}$ and $z_1(t)$, $z_2(t)$ … … … … $z_p(t)$ are the premise variables.

Given a pair of x(t) and u(t), the model dynamics is given as follows:

$$\dot{\boldsymbol{x}}(\boldsymbol{t}) = \sum_{i=1}^{r} \boldsymbol{w}_i\left(\boldsymbol{z}(\boldsymbol{t})\right)(\boldsymbol{A}_i\boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{B}_i\boldsymbol{u}(\boldsymbol{t})) \tag{3}$$

where

$$\boldsymbol{w}_i\left(\boldsymbol{z}(\boldsymbol{t})\right) = \frac{\boldsymbol{\mu}_i(\boldsymbol{z}(\boldsymbol{t}))}{\sum_{k=1}^{r}\boldsymbol{\mu}_k(\boldsymbol{z}(\boldsymbol{t}))}$$

$$\boldsymbol{\mu}_i(\boldsymbol{z}(\boldsymbol{t})) = \prod_{j=1}^{p} \boldsymbol{N}_{ij}\left(\boldsymbol{z}_j(\boldsymbol{t})\right)$$

$N_{ij}(z_j(t))$ is the degree of membership function of $z_j(t)$ in N_{ij}.

Here, $\mu_i(z(t)\geq 0$, for i=1, 2, 3 … … … … r

and $\Sigma^r_{i=1}\ \mu_i(z(t)>0$ for all t.

Therefore, $w_i\ (z(t))\geq 0$

for i=1, 2, 3 … … … … r and $\Sigma^r_{i=1}\ w_i(z(t)=1$.

2. T-S Fuzzy Model of Tunnel Diode Circuit

2.1 State Space Model

Consider the tunnel diode circuit as shown in Figure 2, which is characterized by $i_R=h(v_R)$ [18]. Since there are two storing elements L and C in this circuit, therefore two state variables are required. Choosing $x_1(t)=v_C(t)$ and $x_2(t)=i_L(t)$ as state variables, and u(t)=E as control input, the state space equation of the tunnel diode circuit is given as [18, 19]:

$$\dot{x}_1(t) = \frac{1}{C}\left[-h\left(x_1(t)\right) + x_2(t)\right] \tag{4}$$

$$\dot{x}_2(t) = \frac{1}{L}\left[-x_1(t) - Rx_2(t) + u(t)\right] \tag{5}$$

Assume the circuit parameter u(t)=1.2 V, R=1.5 KΩ, C=2 pF and L=5 μH. Measuring time in nanosecond and the currents $x_2(t)$ and $h(x_1(t))$ in mA [18], the state space model is given by:

$$\dot{x}_1(t) = 0.5\left[-h\left(x_1(t)\right) + x_2(t)\right] \tag{6}$$

$$\dot{x}_2(t) = 0.2\left[-x_1(t) - 1.5x_2(t) + u(t)\right] \tag{7}$$

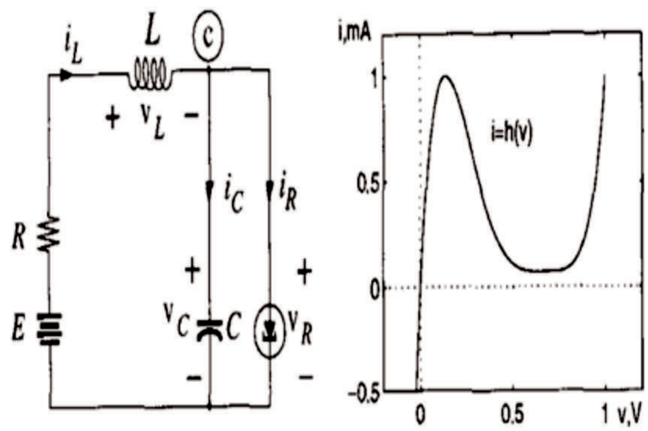
Where h(.) is given by

$h(x_1(t)) = 17.76\ x_1(t) - 103.79\ x_1(t)^2 + 229.62\ x_1(t)^3 - 226.31\ x_1(t)^4 + 83.72\ x_1(t)^5$

Since (6) and (7) are non-linear differential equations which represent the dynamic of the tunnel diode circuit.

2.2 T-S Fuzzy Model

Assume that $x_1(t)\in(-0.5, 0.5)$ and using the following fuzzy sets:



(a) (b)

Figure 2 (a) Tunnel Diode Circuit, (b) Tunnel Diode v_R-i_R Characteristic

$$N_{11} = \frac{x_1(t)^2}{0.25};\ N_{21} = 1 - \frac{x_1(t)^2}{0.25};$$

$$N_{31} = \frac{x_1(t)^4}{0.0625};\ N_{41} = 1 - \frac{x_1(t)^4}{0.0625};$$

Figure 3 shows the membership function of the given fuzzy sets.

Now, the equations (6) and (7) can exactly be represented by the following fuzzy rules:

Rule 1 IF $x_1(t)$ is N_{11}

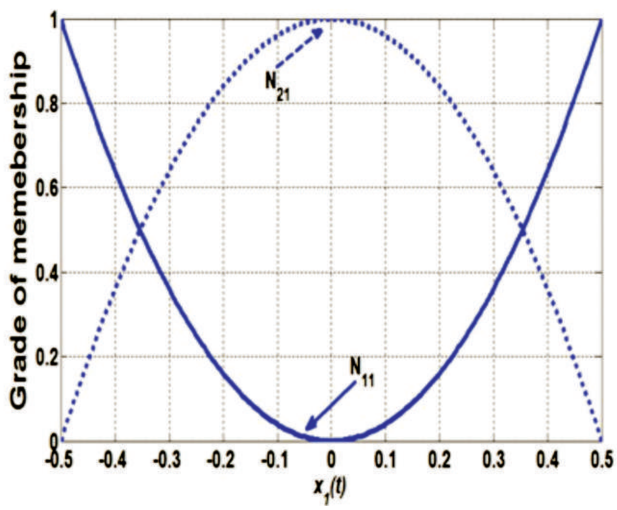
THEN $\dot{x}(t)=A_1x(t)+B_1u(t)$

Rule 2 IF $x_1(t)$ is N_{21}

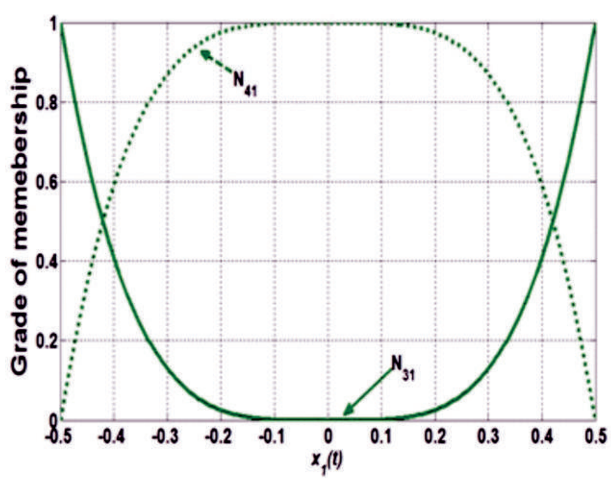
THEN $\dot{x}(t)=A_2x(t)+B_2u(t)$

Rule 3 IF $x_1(t)$ is N_{31}

THEN $\dot{x}(t)=A_3x(t)+B_3u(t)$



(a)



(b)

Figure 3. Membership Function of the Fuzzy System

Rule 4 IF $x_1(t)$ is N_{41}

THEN $\dot{x}(t)=A_4x(t)+B_4u(t)$

Where $A_1 = A_3 = \begin{bmatrix} -0.106875 & 0.5 \\ -0.2 & -0.3 \end{bmatrix}$

$$A_2 = A_4 = \begin{bmatrix} -8.88 & 0.5 \\ -0.2 & -0.3 \end{bmatrix} \text{ and } B_i = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix}.$$

i=1, 2, 3, 4

Given a pair of x(t) and u(t), the model dynamics is given as follows:

$$\dot{x}(t) = \sum_{i=1}^{4} w_i\left(z(t)\right)\left(A_i x(t) + B_i u(t)\right) \qquad (8)$$

Where

$$w_i\left(z(t)\right) = \frac{\mu_i(z(t))}{\sum_{k=1}^{4}\mu_k(z(t))}$$

3. Result

It is observed that the fuzzy system (8) is unstable even though the linear sub-systems are stable. Hence for the steady state analysis of the tunnel diode circuit, it is necessary that the overall fuzzy system must be stable. By employing the Parallel Distributed Compensation (PDC) [20], the fuzzy controller is designed by choosing the feedback gain as:

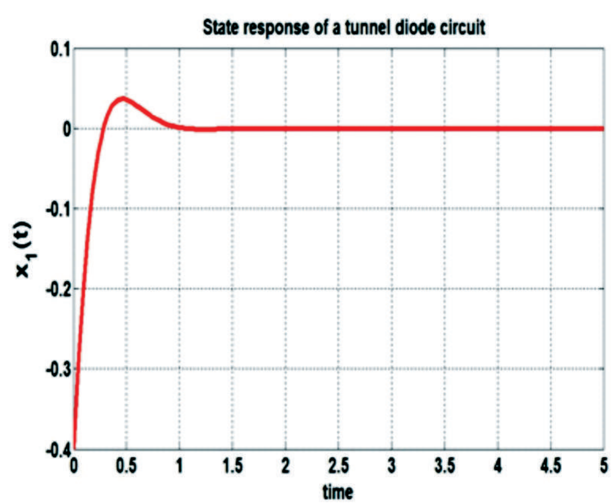
$K_1, K_3=[169.4955 \quad 42.9656]$

$K_2, K_4=[169.344 \quad -0.9]$

Figures 4 and 5 show the stable response of the tunnel diode circuit.

Conclusion

In this paper, the fuzzy modelling of tunnel diode circuit is



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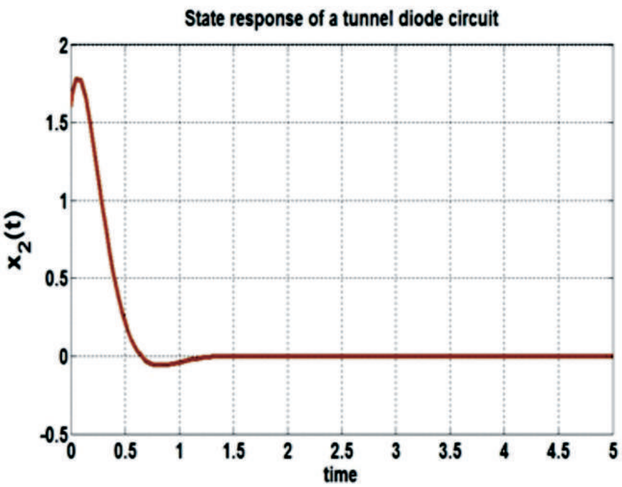
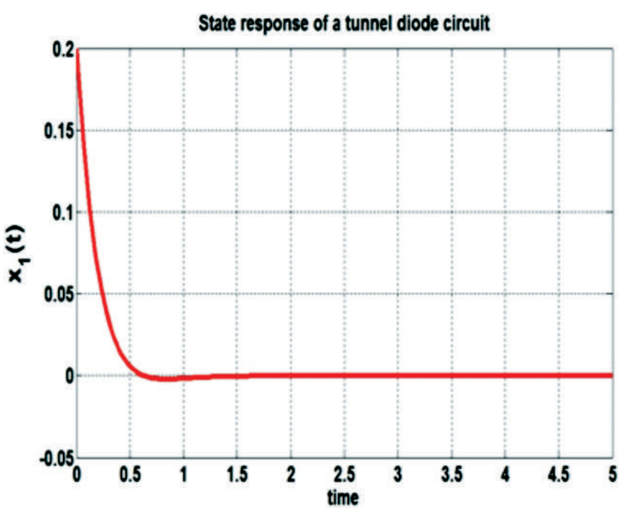
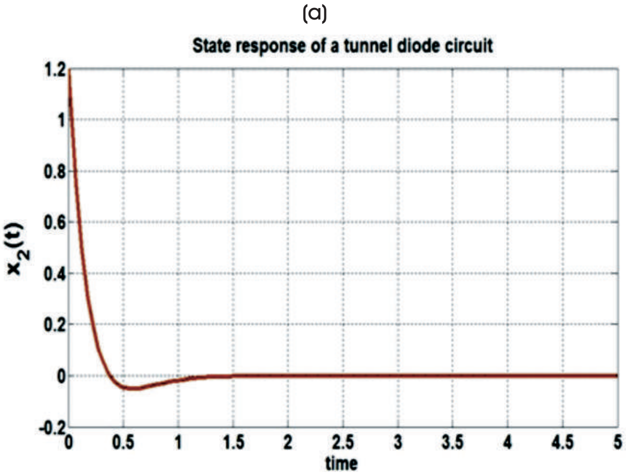


(b)

Figure 4. Fuzzy Model response of a Tunnel Diode Circuit with $x(0)=[-0.4, 1.6]$ (a) $x_1(t)$, (b) $x_2(t)$



(a)



(b)

Figure 5. Fuzzy Model response of a Tunnel Diode Circuit with $x(0)=[0.2, 1.2]$ (a) $x_1(t)$, (b) $x_2(t)$

done which is an example of highly nonlinear system. The resulting linearized model is about negative resistance region of the tunnel diode with less approximation error. It is seen from the simulation result that the response of fuzzy system has fast settling time, less overshoot and stable response with high approximation accuracy.

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